netic field maximum of the pump.

The signal, associated with a $\Delta S_Z = \pm 1$ transition, requires that the signal rf magnetic vector be perpendicular to the large applied dc field. The pump, associated with a $\Delta S_Z = 0$ transition, requires that its magnetic vector be parallel to the dc field. In the cavity geometry employed the signal, and pump rf magnetic fields are parallel to each other. The applied dc magnetic field is oriented at an angle of 45° to the plane of the strip in order to have rf field components satisfying the two transition probability requirements.

Performance

This device was constructed for some initial feasibility tests and its performance should not be taken to indicate the limits of performance which may be expected in a more practical device.

A pumping power of 38 mw was employed, probably less than 1 per cent of this being absorbed by the crystal. The discrepancy arises from the fact that a low pumping cavity Q provides a poor match. The pumping requirements may be materially decreased by better design.

With a gain of 20 db the bandwidth is ≈ 100 kc. Although the material has a line width of 30 mc this inherent bandwidth is not utilized because of the high $Q(\approx 6000)$ of the signal cavity and the associated positive feedback. The regeneration also results in a fairly small stability margin. Both bandwidth and stability would be improved if an appropriate unidirectional slow wave structure was employed.

The effective noise temperature of the entire device including circulator and monitoring equipment was $\sim 150°$K. Most of this noise arose from the excessive, and easily reducible, room temperature losses of the circulator and monitoring equipment. The noise temperature of the actual maser was $< 35°$K compared with the theoretical value of 4°K (obtained from a spin temperature of $-4°$K). The accuracy of the noise measuring apparatus was insufficient to allow identification of the true maser noise.

Nonmechanical Beam Steering by Scattering from Ferrites*

M. S. WHEELER†

Summary—A small aperture radiating circularly polarized energy is loaded with a spherical ferrite to produce an electronic beam directing system. The ferrite is immersed in a static magnetic field which is in general at an oblique angle with the undeflected direction of radiation. It is shown that radiation is principally in the direction of the magnetic field when the polarization is in the negative sense. From symmetry this allows beam deflection with two degrees of freedom.

To consider an application for such a device, it is proposed that this deflection system be used in conical scan. A mechanization is shown which solves the problem in principle, but it is not competitive with present mechanical scanners from the point of view of side lobes, etc.

Introduction

USE is continually being made of two ferrite properties, the Faraday rotation and resonant absorption, to produce the many microwave components used today in waveguides and antennas. When these principles are applied to beam steering in antennas, it would seem that the properties of the ferrite are used in an indirect manner. Alternately, it should be possible to use ferrites directly to produce a deflection as the energy is passed through and over the ferrite. Such an effect was reported by Angelakos and Korman.[1]

It would be required that the ferrite be immersed in a large fraction of the radiated energy, and yet the ferrite must be relatively free from boundaries in such a manner that the energy would be free to change direction. The asymmetry, which is controlled and used to change beam direction, would be the relatively static saturating magnetic field to which direction all of the anisotropic properties of the ferrite are referred.

The advantages of such a beam-steering device are obvious: no mechanically moving parts are necessary. Less control power would be required, low-temperature starting would cease to be troublesome, and higher operating speeds could be achieved with a ferrite as compared to a corresponding mechanical deflection system. However, to be practically useful, it would be required that a ferrite scatterer have low dissipative loss, that it be made relatively reflectionless, and that a sufficient degree of beam perfection be achieved considering cross polarization and extraneous side lobes.

* Manuscript received by the PGMTT, May 20, 1957; revised manuscript received, August 5, 1957.
† Westinghouse Electric Corp., Air Arm Div., Baltimore 3, Md.

[1] D. J. Angelakos and M. M. Korman, "Radiation from ferrite filled apertures," PROC. IRE, vol. 44, pp. 1463–1468; October, 1956.

Mechanization of Controllable Scattering by Ferrite

It is possible to visualize the mechanism for beam deflection from the previous discussion and from consideration of symmetry. Consider a waveguide radiating from an open end. The beam, formed along the waveguide center line, is to deflect symmetrically in azimuth and elevation. This symmetrical deflection, it would appear, could be most easily achieved with a circular waveguide and circular polarization of the incident energy. This condition is not a fundamental requirement, but it does produce a practical system and achieves the required symmetry automatically.

The ferrite would have to be near the end of the waveguide. When too far out of the guide, the ferrite intercepts only a small amount of the energy; when too deep in the guide, beam deflection is prevented by the guide walls. Considering now only uniform static magnetic fields, there would be, in general, two components—one axial which can produce no deflection because of its symmetry, and one transverse which is the only intentional asymmetry and the only source of the expected beam deflection.

The source of this static magnetic field may be a combination of permanent and electromagnets surrounding the ferrite. While it has been found possible to place the magnets internal to the ferrite resulting in very low control power, several practical considerations lead to the placing of the magnets external to the ferrite. In Fig. 1, a typical magnet-ferrite combination is shown from which considerable data have been taken.

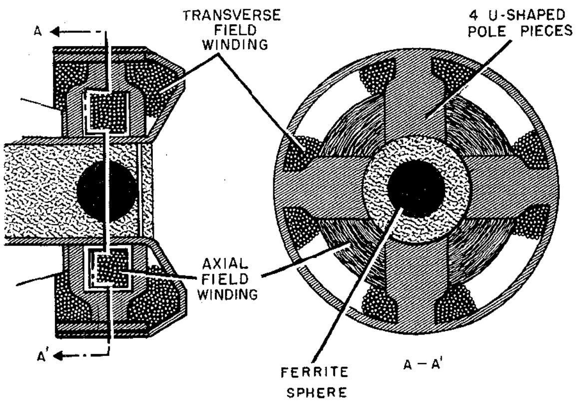

Fig. 1—Magnetic circuit for supplying oblique magnetic field.

An axial-field winding surrounds the end of the waveguide. Covering the axial-field winding in part, are four U-shaped pole pieces which complete the external magnetic circuit for the axial winding and at the same time provide a core upon which the transverse windings are placed. The U-shaped pole pieces thus provide both axial and transverse components of the static magnetic field. The transverse field magnetic circuits in both azimuth and elevation are closed around the periphery of the whole assembly by a soft iron ring. The transverse magnetic field path might be visualized in section A-A' of Fig. 1 as crossing the sphere, dividing in the U-shaped pole pieces, and proceeding to the outer ring where it again divides to return to the opposite pole piece around the outside diameter. The axial-field path might be visualized in the first cut of Fig. 1. As the axial field crosses the sphere, it splits four ways, returning to the opposite side of the sphere through the U-shaped pole pieces which also conduct the transverse field. The total static magnetic field in the ferrite is thus an oblique field which is the vector sum of three mutually perpendicular magnetic fields. The ferrite itself is shown imbedded in a teflon support which centers the ferrite and maintains propagation in the necked-down waveguide.

Dipole Model for Small Sphere in a Plane Wave

Unfortunately, the deflection system as described above has not proved susceptible to analysis. It is instructive, however, to apply the work of Berk and Lengyel[2] for small spheres in plane waves. A coordinate system is shown in Fig. 2 where the static magnetic field is along the Z axis, and propagation is along the Z' axis which is tipped at an angle θ with respect to Z.

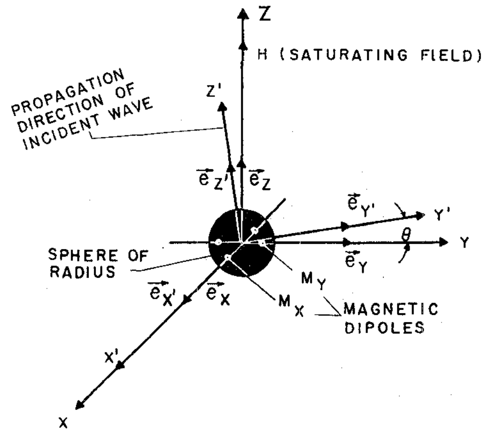

Fig. 2—Coordinate system for a small sphere of ferrite in a plane circularly polarized wave.

Here the $\vec{e}$ vectors are unitary along their respective axes. An elliptically polarized incident wave is assumed propagating in the Z' direction—$h=(h_{0x'}\vec{e}_{x'}+jh_{0y'}\vec{e}_{y'})$ exp $(j\omega t-\Gamma Z')$. Then in the notation of Berk and Lengyel, a field is found external to the sphere consisting of the original field plus a dipole field. The sphere will be assumed saturated in the Z direction:

$$H = \vec{e}_x h_{0x}' + \vec{e}_y j\cos\theta h_{0y}' + \vec{e}_z j\sin\theta h_{0y}' + (D_{1_{x'}} - D_{2_{y'}})\left[\vec{e}_x\frac{r^2-3x^2}{r^5} - \vec{e}_y\frac{3xy}{r^5} - \vec{e}_z\frac{3xz}{r^5}\right] - (D_{1_{y'}} + D_{2_{x'}})\left[\vec{e}_x\frac{3xy}{r^5} - \vec{e}_y\frac{r^2-3y^2}{r^5} + \vec{e}_z\frac{3yz}{r^5}\right],$$

[2] A. D. Berk and B. A. Lengyel, "Magnetic field in small ferrite bodies with applications to microwave cavities containing such bodies," Proc. IRE, vol. 43, pp. 1587–1591; November, 1955.

where

$$D_{1_x'} = \left[\frac{3(\mu + 2)}{(\mu + 2)^2 - \propto^2} - 1\right] h_{0x'} a^3,$$

$$D_{2_x'} = \left[\frac{-3\propto}{(\mu + 2)^2 - \propto^2}\right] j h_{0x'} a^3$$

$$D_{1_y'} = \left[\frac{3(\mu + 2)}{(\mu + 2)^2 - \propto^2} - 1\right] j h_{0y'} a^3,$$

$$D_{2_y'} = \left[\frac{3\propto}{(\mu + 2)^2 - \propto^2}\right] h_{0y'} a^3.$$

If now three mutually perpendicular dipoles are assumed at the sphere on the xyz coordinates of magnitudes m_x, m_y, and m_z, it can be shown that the scattered portions of the field outside the ferrite can be reproduced by letting

$$m_x \frac{1}{4\pi\mu_0 h_{0x'}} = -\frac{(\mu^2 - \propto^2 + \mu - 2) + 3\propto\cos\theta\dfrac{h_{0y'}}{h_{0x'}}}{(\mu + 2)^2 - \propto^2}$$

$$m_y \frac{1}{4\pi\mu_0 h_{0x'}} = -j\frac{(\mu^2 - \propto^2 + \mu - 2)\cos\theta\dfrac{h_{0y'}}{h_{0x'}} + 3\propto}{(\mu + 2)^2 - \propto^2}$$

$$m_z = 0.$$

From this dipole orientation, it can be seen that the direction of maximum scattered energy is along the axis of the magnetic field. Then, if means were found to block the overriding plane incident wave, a beam would have been formed principally along the axis of the static magnetic field. There are three other significant observations that can be made regarding the scattered energy. First, with an incident circularly polarized wave ($h_{0y'} = \pm h_{0x'}$) and no deflection ($\theta = 0$), the scattered wave is circularly polarized. As deflection is produced, however, an increasing degree of depolarization must be expected. Second, as the functions are even in θ, the deflection left or right could be expected with equal amplitudes when the transverse field is reversed. Third, for typical X-band ferrites below magnetic saturation,[3] the dipole strengths, m_x and m_y, are greater for the negative sense of circular polarization ($h_{0y'} = h_{0x'}$). These three points are found generally in the measurements which will be described in the following section.

Measurements on a Spherical Ferrite Scatterer

The preceding analysis has shown that the field around a small sphere in a saturating magnetic field and a circularly polarized plane wave is made up of the incident wave plus a dipole field radiating principally along the magnetic field. In practice the incident wave must be blocked by a suitable boundary in order to achieve a useful deflection. Therefore, the sphere is no longer in a plane wave. Further, experiment finds that useful spheres are not small compared with the wavelength in the ferrite. For these reasons, the measurements which will be described here agree only qualitatively with the calculations.

Measurements have been made on cylindrical and spherical ferrite samples. The spherical ferrites, however, are definitely preferred for minimum depolarization for a given deflection. The transmitted power is measured at the peak of the beam in whatever direction it falls. It is referred to power transmitted with no ferrite at all in the system. Thus, power loss is made up of dissipation in the ferrite, reflection from the ferrite, and spreading of the transmitted beam. The incident and detected power are both circularly polarized. The beam deflection is measured by placing the detector at 45° with respect to the undeflected beam and measuring db change in transmitted power upon reversing the transverse field. This is found much more easily than measuring the angle of the broad peak involved. Although the normal input to the radiator is circularly polarized, standing wave ratios are measured with linear polarization along two axis—one with the magnetic rf field parallel to the transverse static field, and one with the rf field perpendicular to the transverse static field. The degree of depolarization (cross polarization) is measured to the same reference as the normal polarization by reversing the polarization sense of the detector.

The data in Fig. 3(a) were taken at 9200 mc with a 0.350-inch-diameter General Ceramics R-1 sphere supported in teflon at the end of a 0.582-inch-diameter waveguide as shown in Fig. 1. It should be mentioned that the extension of the dielectric beyond the sphere affects both the standing wave ratio and the beam deflection performance. In this case, the teflon is extended about 0.125 inch beyond the end of the sphere. In Fig. 3(a), the maximum db difference shown corresponds to about ±15° of beam deflection. While this does not represent the maximum possible, the curves show typically a critical value of axial field giving maximum transmitted power, greater deflection with negative circular polarization than with positive, two symmetrical axial fields of minimum standing wave ratio, and an increasing depolarization of the fields of largest beam deflection.

Now the data in Fig. 3(a) were taken at constant transverse magnetic field. For a better comparison with the calculated dipole strengths m_x and m_y, the ratio of axial to transverse field was held constant to maintain the static field at 30° to the waveguide center line. In Fig. 3(b), measured deflections and cross polarization are compared with calculated pole strengths from ferrite characteristics given for R-1 by Spencer and Le Craw. From the previous equations, dipole strengths driving the rf fields along the 30° static magnetic field axis in the forward sense of polarization are $(m_x - jm_y)/2$. Similarly, $(m_x + jm_y)/2$ in Fig. 3(b) represents energy along this axis in the cross-polarized sense. It can be seen that

[3] See the characteristics of R. E. LeCraw and E. G. Spencer, "Tensor permeabilities of ferrites below magnetic saturation," 1956 IRE Convention Record, pt. 5, pp. 66–74.

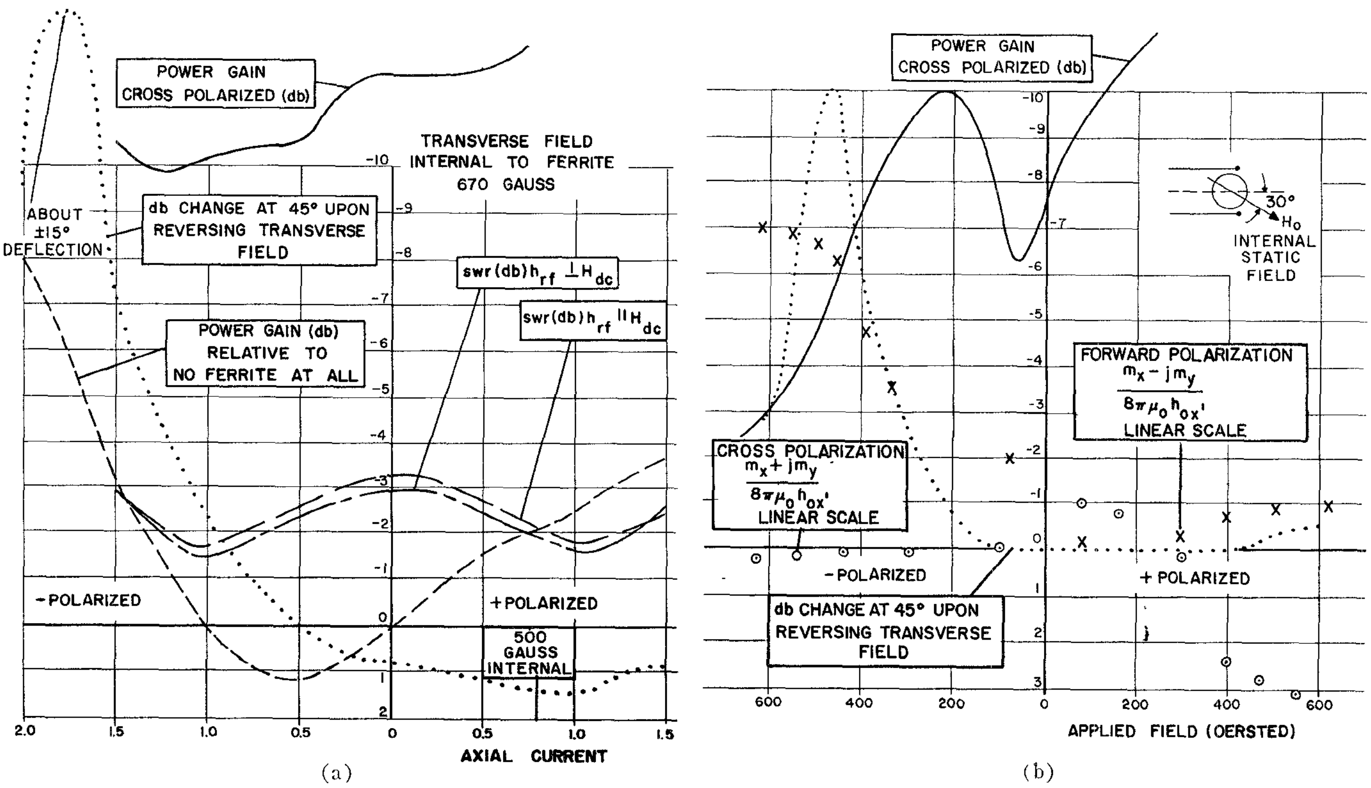

Fig. 3—(a) Deflection characteristic of 0.350-inch diameter sphere. (b) Beam deflection for static field at 30°. The X-data points indicate calculated dipole strengths driving circularly polarized energy in the direction of the static magnetic field with the same sense of polarization as the incident wave. The ⊙-data points are calculated dipole strengths in the reverse sense of polarization. The other curves are experimental.

the deflection is great where $(m_x - jm_y)/2$ is large and that the deflection is in the anticipated direction. Further, $(m_x + jm_y)/2$ predicts the cross-polarized component, except for the region of positive polarization where one would expect to measure severe depolarization. Thus the rf dipole representation of the problem gives a very nice qualitative picture of the beam deflection mechanism. Further, it is possible to judge the usefulness of any particular ferrite in this application if the microwave permeabilities are known.

Typical of most ferrite devices, the deflection direction is nonreciprocal. That is, a characteristic curve is preserved when the direction of propagation is reversed only if the sense of polarization is also reversed. This characteristic is, of course, no problem in a one-way antenna. In a two-way antenna, on the other hand, such as is used for radar, this may be an advantage or a disadvantage, depending upon the application. If this principle were applied to radar conical scanning, for example, the reciprocity problem could be solved with the transmitter and receiver coupled to a circular waveguide in space quadrature. The transmitted and received power would then have the opposite sense of circular polarization, and the transmitted and received beam would both lie in the same direction. Such a radar, though circularly polarized, would have the same rain rejection and target characteristics as if it were linearly polarized.

Application of a Ferrite Scatterer to Conical Scanning

While there are limited applications for the wide beam deflection system as it has been described, the original objective of this development was to produce conical scanning for radar by ferrite means. The required mechanical motion of conventional scanners is relatively rapid, and low-temperature operation is a recurrent problem. Further, conical scanning is a simplified problem as it is only a one-degree-of-freedom application.

The chief difficulty is this: in conical scanning from a parabolic reflector, an effective center of radiation is displaced from the axis of the parabola and rotated at a constant radius. The problem then becomes one of converting a deflected beam from the ferrite scatterer into a displaced center of radiation. Accordingly, various splash-plate and dielectric lens combinations have been tried in an effort to produce this effect.

Although this problem is far from being solved, one of the most effective combinations that has been found to the present is the feed shown in Fig. 4. Here power is deflected at the ferrite and reflected toward a parabolic antenna dish at the far left (not shown on the drawing). Relative sizes are preserved, and the ferrite diameter is 0.350 inch, which gives its approximate size. A complete physical description is not given, however, as the device is rather imperfect. For example, in order to achieve a 1-db conical scan crossover, the depolarized power is

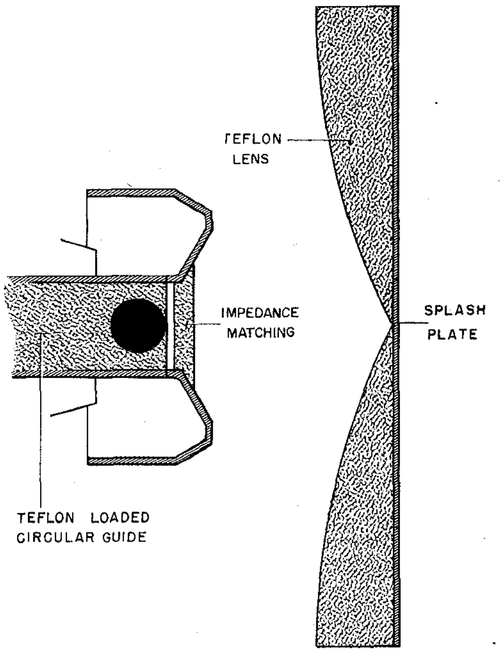

Fig. 4—Ferrite application to a feed for a conical scanning antenna.

only 10 db below the normal polarization. Further side lobes are up to 15 db below the peak of the beam, and antenna efficiency is low. Nevertheless, the device demonstrates an application which could produce a conical scanner in principle in all respects including the problem of reciprocity in the ferrite.

Conclusion

It has been shown that a small sphere immersed in an oblique uniform magnetic field and an rf circularly polarized plane wave will scatter electromagnetic energy principally along the magnetic field. It is demonstrated that this principle can be used to direct a wide radiated beam at the expense of some depolarization of the incident wave. Finally, it has been shown as an example that this could be applied to the problem of conical scanning although a scanner has not been made which could compete with present mechanical scanners.

Acknowledgment

Most of the measurements described were made by R. Cox of the Westinghouse Air Arm Division.

A Ferrite Boundary-Value Problem in a Rectangular Waveguide*

C. B. SHARPE† AND D. S. HEIM†

Summary—A solution is obtained for the electric field at the air-ferrite interface ($z=0$) in a rectangular waveguide filled with ferrite in the semi-infinite half ($z>0$) and magnetized in the direction of the electric field. The field is expressed in terms of a Neumann series obtained by iteration of a singular integral equation which satisfies the boundary conditions at the interface. The equivalent circuit for the junction is also presented.

Introduction

THE mathematical difficulties which are encountered in the solution of many boundary value problems involving gyromagnetic media have been pointed out by several authors.[1-3] The formulation of these problems is usually straightforward but the imposition of the boundary conditions at the isotropic-to-anisotropic interface frequently makes them intractable. The one discussed here, in which the anisotropic media is a semi-infinite slab of ferrite filling a rectangular waveguide, appears to present some of the essential difficulties common to the solution of many such problems.

Referring to Fig. 1, we consider an infinite rectangular waveguide which is filled with a ferrite medium for $z>0$ and air for $z<0$. The ferrite region is magnetized in the y direction with an internal field H. A TE_{10} wave is incident from the left at the air-ferrite interface ($z=0$). The problem is to determine the electric and magnetic fields at the interface and the equivalent circuit for the junction. The ferrite medium is assumed to be lossless and characterized by a tensor permeability

$$(\mu) = \begin{pmatrix} \mu & 0 & j\kappa \\ 0 & \mu_0 & 0 \\ -j\kappa & 0 & \mu \end{pmatrix}$$

where

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† Electronic Defense Group, Dept. of Elec. Eng., Univ. of Michigan, Ann Arbor, Mich.

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[3] H. Suhl and L. R. Walker, "Topics in guided wave propagation through gyromagnetic media," *Bell Sys. Tech. J.*, vol. 33, Part I, pp. 579–659; May, 1954, Part II, pp. 939–986; July, 1954, and Part III, pp. 1133–1194; September, 1954.